1116-03-1649Joachim Mueller-Theys* (mueller-theys@gmx.de). Does the Consistency Sentence Really
State Consistency?

The Second Incompleteness Theorem actually makes 2 assertions:

(1) $\operatorname{Con}_{\Sigma}$ states that Σ is consistent;

(2) $\Sigma \not\vdash \operatorname{Con}_{\Sigma}$ if $\Sigma \supseteq \Sigma_{\operatorname{PA}}$ is consistent.

(1) had no explicit definiens.

If (1) is—as the definiendum, lacking another statement of place, suggests—related to (the theory of) Σ , then, as we will show, (2) implies non (1), whence (1) & (2) becomes a contradiction in terms. In addition, the generalisation: κ states consistency, cannot be fulfilled at all.

If Σ is decidable, (1) becomes true in Th (\mathcal{N}), the deductively inaccessible theory of arithmetics.

More innately, κ states that Σ is consistent :iff $\Sigma \not\vdash \kappa$. Consequently, if Σ is consistent, all of the then existing κ , unprovable from Σ , state this, and, if Σ is inconsistent, no κ states that Σ is consistent. If (1) is interpreted in this way, (1) follows from (2), but Con Σ is not distinguished from any other $\Sigma \not\vdash \kappa$.

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