Levent Alpoge* (lalpoge@math.princeton.edu). Square-root cancellation for the signs of Latin squares (i.e., why the Alon-Tarsi conjecture is hard).
Let $L(n)$ be the number of Latin squares of order $n$, and let $L^{\text {even }}(n)$ and $L^{\text {odd }}(n)$ be the number of even and odd such squares, so that $L(n)=L^{\text {even }}(n)+L^{\text {odd }}(n)$. The Alon-Tarsi conjecture states that $L^{\text {even }}(n) \neq L^{\text {odd }}(n)$ when $n$ is even (when $n$ is odd the two are equal for very simple reasons). We prove that $\left|L^{\text {even }}(n)-L^{\text {odd }}(n)\right| \leq L(n)^{\frac{1}{2}+o(1)}$, thus establishing the conjecture that the number of even and odd Latin squares, while conjecturally not equal in even dimensions, are in fact equal to leading order asymptotically. The proof is actually very short: we apply a differential operator to an exponential integral over $\mathrm{SU}(n)$ and calculate what results in two different ways. The method is inspired by a result of Kumar-Landsberg. (Received September 17, 2015)

