1116-20-1981Anton Lukyanenko* (lukyanen@umich.edu) and Joseph Vandehey. Carnot vs Siegel:
Diophantine approximation in the Heisenberg group.

In 1966, W. Schmidt used a game theory argument to prove that the set of badly approximable real numbers has full Hausdorff dimension, despite having measure zero. The argument was extended in 2010 by McMullen to the set of points D(G) in the boundary of a hyperbolic space that are occluded by a collection of horoballs associated to a non-uniform lattice G.

Interpreting McMullen's result for the Heisenberg group H^1 (a boundary of the complex hyperbolic plane), we show that the set of badly approximable points in the Siegel model of H^1 has full Hausdorff dimension. We then compare Diophantine approximation in the Carnot and Siegel models of H^n , finding a difference in both the critical Diophantine exponent and the structure of badly approximable points (the latter via a Schmidt games result).

In particular, we obtain a correspondence between the set D(G) (for G the Picard modular group), the set of points with bounded continued fraction digits, and set of badly approximable points in the Siegel model of H^1 . This provides the first strong link between Heisenberg continued fractions and complex hyperbolic geometry. (Received September 21, 2015)