Let $K$ be a field and $f : \mathbb{P}^N \rightarrow \mathbb{P}^N$ a morphism. There is a natural conjugation action on the space of such morphisms by elements of the projective linear group $\text{PGL}_{N+1}$. The group of automorphisms, or stabilizer group, of a given $f$ for this action is known to be a finite group. In this talk, we discuss a mainly computational problem concerning automorphism groups: Given a finite subgroup of $\text{PGL}_{N+1}$ determine endomorphisms of $\mathbb{P}^N$ with that group as subgroup of its automorphism group. In particular, we show that every finite subgroup occurs infinitely often and discuss some associated rationality problems. (Received September 21, 2015)