We describe a general method for presentations of colimit modules of functors into module categories. This is applied to the Bar-Natan functor, which is defined on a category of surfaces embedded in a 3-manifold $M$ with morphisms defined by certain 3-manifolds embedded in $M \times [0,1]$ and takes values in a category of modules defined from a commutative Frobenius algebra. The colimit of the Bar-Natan functor is the Bar-Natan module of $M$. Our approach naturally leads to the definition of the tunneling graph of $M$, which contains the geometric data necessary to deduce the structure of the Bar-Natan module. (Received March 07, 2011)