Let $\mathcal{A}$ be the unital associative algebra over $\mathbb{C}$ with generators $x, y, z$ and relations $xy + yx = 2z, yz + zy = 2x$ and $zx + xz = 2y$. We find the finite-dimensional irreducible $\mathcal{A}$-modules and show that $x, y, z$ act on these modules as bipartite or almost bipartite Leonard triples. We define an operator $s$ on finite-dimensional $\mathfrak{sl}_2$-modules that gives them an $\mathcal{A}$-structure.

Let $d$ denote a nonnegative integer and let $Q_d$ denote the graph of the $d$-dimensional hypercube. It is known that the Terwilliger algebra of $Q_d$ has an $\mathfrak{sl}_2$-module structure. When $d$ is even, we show that applying $s$ to the Terwilliger algebra of $Q_d$ produces the Terwilliger algebra of the alternate $Q$-polynomial structure of $Q_d$. When $D$ is odd, we show that applying $s$ to the Terwilliger algebra of $Q_d$ produces the Terwilliger algebra of the antipodal quotient of $Q_d$. (Received August 22, 2011)