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Michael Hugh Knowles* (mhk@mhknowles.net), Rua Miradouro, 45, Sion, Belo Horizonte, MG 30310-640, Brazil. *Does the Banach-Tarski Paradox have an Evil Twin?!*

New theorem on bijections: if the pre-image and image sets SP and SI of a bijection $B(SP,SI)$ have an element EC in common, then one can construct a bijection from the pre-image set with EC removed onto the image set with EC removed, i.e. $B^*(SP-\{EC\},SI-\{EC\})$. Simple proof: if EC is identity subbjected onto EC under B , this identity subbjection is removed, trivially constructing the desired bijection, $B^*(SP-\{EC\},SI-\{EC\})$. But, if EC is subbjected onto some other image element EI , then some other pre-image element EP is subbjected onto EC . We switch the pre-image EC and EP , preserving bijectivity. This yields a bijection $B'(SP,SI)$, with EP subbjected onto EI , and EC identity subbjected onto EC . Again this identity subbjection from EC onto EC is removed from $B'(SP,SI)$, trivially constructing the desired $B^*(SP-\{EC\},SI-\{EC\})$. We apply this theorem to a “Dedekind-infinite bijection” (a bijection from a set SD onto a proper subset of itself, showing that SD is Dedekind-infinite) so as to remove all common elements. We obtain a “Paradoxical Bijection” from a non-empty set onto the empty set, the Evil Twin of the Banach-Tarski Paradox, and a challenging new paradox for the community. (Received December 21, 2011)