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**Jingfen Lan** and **Linyuan Lu\*** (lu@math.sc.edu), Columbia, SC 29208. *Diameters of Graphs with Spectral Radius at most  $\frac{3}{2}\sqrt{2}$ .*

The spectral radius  $\rho(G)$  of a graph  $G$  is the largest eigenvalue of its adjacency matrix. Woo and Neumaier discovered that a connected graph  $G$  with  $\rho(G) \leq 3/2\sqrt{2}$  is either a dagger, an open quipu, or a closed quipu. In this paper, we proved the following results. For any open quipu  $G$  on  $n$  vertices ( $n \geq 6$ ) with spectral radius less than  $3/2\sqrt{2}$ , its diameter  $D(G)$  satisfies  $D(G) \geq (2n - 4)/3$ . This bound is tight. For any closed quipu  $G$  on  $n$  vertices ( $n \geq 13$ ) with spectral radius less than  $3/2\sqrt{2}$ , its diameter  $D(G)$  satisfies  $\frac{n}{3} < D(G) \leq \frac{2n-2}{3}$ . The upper bound is tight while the lower bound is asymptotically tight. Let  $G_{n,D}^{min}$  be a graph with minimal spectral radius among all connected graphs on  $n$  vertices with diameter  $D$ . For  $n \geq 13$  and  $D \in [\frac{n}{2}, \frac{2n-7}{3}]$ , we proved that  $G_{n,D}^{min}$  is the graph obtained by attaching two paths of length  $D - \lfloor \frac{n}{2} \rfloor$  and  $D - \lceil \frac{n}{2} \rceil$  to a pair of antipodal vertices of the even cycle  $C_{2(n-D)}$ . Thus we settled a conjecture of Cioab-van Dam-Koolen-Lee, who previously proved a special case  $D = \frac{n+e}{2}$  for  $e = 1, 2, 3, 4$ . (Received January 17, 2012)