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**Thomas Dinitz, Matthew Hartman and Jenya Soprunova\*** (soprunova@math.kent.edu),  
E. Summit st., Kent, OH 44242. *Tropical determinant on the Birkhoff polytope.*

We start with the following combinatorial problem: Given a Rubik's cube, solve it by peeling off and replacing the stickers. How many stickers would you need to peel off and replace in the worst case scenario?

This problem, translated into the language of matrices, generalizes to the following question: Given positive integers  $m$  and  $n$ , find the sharp lower bound  $L(m, n)$  on the tropical determinant of integer doubly-stochastic  $n$  by  $n$  matrices whose row and column sums are equal to  $m$ . Hence the initial problem boils down to minimizing the tropical determinant over the integer points of the Birkhoff polytope. We provide a complete solution to this problem. The question that we answer can also be interpreted as an integer tropical version of the van der Waerden conjecture about permanents. (Received January 17, 2012)