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Joshua N Cooper* (cooper@math.sc.edu), 1523 Greene St., LeConte College, USC, Columbia, SC 29208, and **Anna Kirkpatrick**. *Critical Sets for Graph Coloring*. Preliminary report.

Recently, Civario, McGuire, and Tugemann surprised the mathematics-of-Sudoku community by announcing a (computer-assisted) proof that a fair puzzle must have at least 17 givens (a.k.a. clues). One may interpret this as a result about “critical sets” for proper vertex colorings of the “Sudoku graph” Sud . In particular, given a coloring c of a graph G , we say that $S \subseteq V(G)$ is “determining” for c if the only proper vertex coloring extending $c|_S$ to all of $V(G)$ is c itself; S is said to be “critical” if it is determining and minimally so. Given a coloring c of G , we may define $\text{scs}(G, c)$ to be the size of the smallest critical set and $\text{lcs}(G, c)$ to be the size of the largest; then $\underline{*}(G)$ is the smallest value of $*(G)$ and $\overline{*}(G)$ is the largest value of $*(G)$ over all proper vertex colorings c of G , where $*$ = scs or lcs . Then the aforementioned result is the statement that $\underline{\text{scs}}(\text{Sud}) = 17$. Furthermore, previous work on critical sets for Latin squares has shed some light on the values of these four parameters for $K_n \square K_n$.

We discuss the matter of computing these parameters for several graph classes. Open questions abound, and we give several directions for future work. (Received January 18, 2012)