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For the real line  $R$  a function  $F : R \times R \rightarrow R$  is universal iff for any  $f$  of the same type there is  $g : R \rightarrow R$  such that  $f(x, y) = F(g(x), g(y))$  for all  $x, y \in R$ .

Sierpinski asked in the Scottish book, problem 132, if there exists a Borel function  $F(x, y)$  which is universal. He had shown that the answer is yes assuming the continuum hypothesis. If Martin's Axiom is true, then there is a universal function of Baire class 2. A universal function cannot be of Baire class 1.

Here we show that it is consistent that for each  $\alpha$  with  $2 < \alpha < \omega_1$  there is a universal function of class  $\alpha$  but none of class  $\beta < \alpha$ . We show that it is consistent with ZFC that there is no universal function Borel or not, and we show that it is consistent that there is a universal function but no Borel universal function. We also prove some results concerning higher order universal functions. (Received June 23, 2011)