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Katsuya Miyake* (miyakek@bz-csp.tepm.jp). *The twists of Hessian elliptic curves over splitting fields of cubic polynomials and the related elliptic 3-folds.*

We give our recent results on the two-parameter family of algebraic curves $\tilde{H}(\mu, t)$ which are twists of Hesse's elliptic curves $U^3 + V^3 + W^3 = 3\mu UVW$ over the splitting fields $\tilde{K}_{\mu, t}$ over $\mathbb{Q}(\mu, t)$ of the cubic generic polynomial $R(t; X) := X^3 + tX + t$ for the symmetric group of degree 3. The twist is defined over $\mathbb{Q}(\mu, t)$ and of genus 1 if a few specific values of the parameters μ and t are excluded. It may not, however, have any rational points over the base field. Here we give a necessary and sufficient condition for the twist to have a rational point over $\mathbb{Q}(\mu, t)$. In the case where the essential part of our sufficient condition is satisfied, we are able to give an elliptic 3-fold over an affine plane from $\tilde{H}(\mu, t)$. If we restrict ourselves to the case where $\tilde{K}_{\mu, t}$ is a cyclic cubic extension of $\mathbb{Q}(\mu, t)$ we also have a necessary and sufficient condition for the twist $\tilde{H}(\mu, t)$ to have a rational point over $\mathbb{Q}(\mu, t)$, and give another elliptic 3-fold over an elliptic surface which comes out of the essential part of the sufficient condition. (Received December 02, 2011)