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Jacob Fox and **Po-Shen Loh*** (ploh@cmu.edu), Department of Mathematical Sciences, Carnegie Mellon University, Pittsburgh, PA 15213. *On a problem of Erdős and Rothschild on edges in triangles.*

Erdős and Rothschild asked to estimate the maximum number, denoted by $h(n, c)$, such that every n -vertex graph with at least cn^2 edges, each of which is contained in at least one triangle, must contain an edge that is in at least $h(n, c)$ triangles. In particular, Erdős asked in 1987 to determine whether for every $c > 0$ there is $\epsilon > 0$ such that $h(n, c) > n^\epsilon$ for all sufficiently large n . We prove that $h(n, c) = n^{O(1/\log \log n)}$ for every fixed $c < 1/4$. This gives a negative answer to the question of Erdős, and is best possible in terms of the range for c , as it is known that every n -vertex graph with more than $n^2/4$ edges contains an edge that is in at least $n/6$ triangles.

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