

1073-05-51

Jingfen Lan* (a-za1107@163.com), 1907 Wheat St. Apt.B, Columbia, SC 29205, and **Linyuan Lu** (lu@math.sc.edu) and **Lingsheng Shi** (lshi@math.tsinghua.edu.cn). *Graphs with Diameter $n-e$ Minimizing the Spectral Radius.*

For a fixed integer $e \geq 1$, let $G_{n,n-e}^{min}$ be a graph with minimal spectral radius among all connected graphs on n vertices with diameter $n - e$. For $e = 1, 2, 3, 4, 5$, $G_{n,n-e}^{min}$ were determined before. Then Cioabă-van Dam-Koolen-Lee conjectured for fixed $e \geq 6$ that $G_{n,n-e}^{min}$ is in the family $\mathfrak{P}_{n,e} = \{P_{2,1,\dots,1,2,n-e+1}^{2,m_2,\dots,m_{e-4},n-e-2} \mid 2 < m_2 < \dots < m_{e-4} < n - e - 2\}$. We settle their conjecture positively here.

Let $T_{(k_1,k_2,\dots,k_{e-4})} = P_{2,1,\dots,1,2,n-e+1}^{2,m_2,\dots,m_{e-4},n-e-2}$ with $k_i = m_{i+1} - m_i - 1$, for $1 \leq i \leq e - 4$, here $m_1 = 2$ and $m_{e-3} = n - e - 2$. Let $s = \frac{n-6}{e-4} - 2 = \frac{\sum_{i=1}^{e-4} k_i + 2}{e-4}$. For $e \geq 6$ and sufficiently large n , we proved that $G_{n,n-e}^{min}$ must be one of the trees $T_{(k_1,k_2,\dots,k_{e-4})}$ with the parameters satisfying $\lfloor s \rfloor - 1 \leq k_j \leq \lfloor s \rfloor \leq k_i \leq \lfloor s \rfloor + 1$ and $0 \leq k_i - k_j \leq 2$ for $j = 1, e - 4$ and $i = 2, \dots, e - 5$; $|k_i - k_j| \leq 1$ for $2 \leq i, j \leq e - 5$. These results are best possible as shown by cases $e = 6, 7, 8$, where $G_{n,n-e}^{min}$ are completely determined. Moreover, if $n - 6$ is divisible by $e - 4$ and n is sufficiently large, then $G_{n,e}^{min} = T_{(s-1,s,s,\dots,s,s-1)}$. (Received July 21, 2011)