

1073-05-69

John Goldwasser* (jgoldwas@math.wvu.edu) and **John Talbot**. *Vertex Ramsey problems in the hypercube.*

We think of the vertices of the n -cube as the set of all subsets of $1, 2, 3, \dots, n$. We say a subset S of the vertices of the d -cube is t -cube-Ramsey if for sufficiently large n , whenever the vertices of the n -cube are colored with t colors, there must be an embedded monochromatic copy of S . If all sets in S have the same size, then it follows immediately from Ramsey's theorem that S is t -cube-Ramsey, for all t . If S contains sets of different parities then it is not 2-cube-Ramsey, because of the all odd size vertices red and all even size vertices blue coloring. In general it is quite difficult to determine if a set S is t -cube-Ramsey. We determine which sets S which are the union of two or three vertex disjoint cliques are 2-cube-Ramsey. We use the Lovasz local lemma to show that no set which is the union of at least 40 vertex disjoint cliques can be 2-cube-Ramsey. We say a t -coloring of the vertices of the d -cube is layered if all the vertices of the same size get the same color. A key ingredient in our proofs is the following: For each positive integer d , there exists a positive integer N such that for each n greater than N , in every t -coloring of the vertices of the n -cube there is an embedded copy of a d -cube with a layered coloring (Received July 26, 2011)