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Kevin L James* (kevja@clemson.edu), BOX 340975, Clemson, SC 29634-0975. *Prime Distribution and Elliptic Curves.*

Number theorists have long been interested in the distribution of prime numbers. The celebrated prime number theorem gives an asymptotic for the number $\pi(X)$ of primes up to any real number X , namely $\pi(X) \sim \frac{X}{\log X}$. There are many refinements of this, such as an asymptotic form of Dirichlet's theorem on primes in arithmetic progressions. The crowning achievement in this direction is the Chebotaryov Density Theorem. Some proposed refinements of this theory related to elliptic curves are the Sato-Tate conjecture recently proved in many cases by Richard Taylor and the Lang-Trotter conjecture.

We briefly review the Lang-Trotter conjecture. Let $E : y^2 = x^3 + Ax + B$ be an elliptic curve over \mathbb{Q} and consider its reduction modulo p . Hasse's theorem says that the number of points on this curve over \mathbb{F}_p is within $2\sqrt{p}$ of $p + 1$. We define $a_E(p) = p + 1 - \#E(\mathbb{F}_p)$. For an integer r and a curve E as above, Lang and Trotter have conjectured that $\#\{p < X \mid a_E(p) = r\} \sim C_{E,r} \frac{\sqrt{X}}{\log X}$, where $C_{E,r}$ is an explicit constant.

In this talk, we will review the results and conjectures mentioned above and their generalizations to number fields, and we will discuss some recent results in these directions. (Received July 19, 2011)