

1080-03-365

Timothy McNicholl*, timothy.h.mcnicholl@gmail.com. *Local Connectivity, Provability, and Computability.*

A topological space X is *locally connected* if, whenever U is a neighborhood of a point p of X , U contains a connected neighborhood V that contains p . We will discuss the role of effective local connectivity in the proofs of certain classical theorems on space-filling curves and extensions of conformal maps. We will then consider effective and ineffective renditions of some of these theorems and the possibilities for their reverse math strength. More specifically, we will discuss the following results:

- 1) There is a computably compact planar Peano continuum that is not the image of a computable map on $[0,1]$.
- 2) Every effectively locally connected and computably compact Euclidean Peano continuum is the image of a computable map on $[0,1]$.
- 3) It is well-known that the image of a continuous map on $[0,1]$ is locally connected, and the proof is fairly easy. However, there is a computable map on $[0,1]$ whose image is not effectively locally connected. Thus, this direction of the Hahn-Mazurkiewicz theorem can not be proven in BISH or in RCA_0 . We show that over RCA_0 it is equivalent to ACA_0 (arithmetic comprehension). (Received January 31, 2012)