One way to generalize classical electrodynamics is to introduce nonlinear constitutive equations consistent with the Lorentz symmetry. While the usual, linear Maxwell equations are incompatible with a Galilean limit obtained by taking the light speed to infinity, there are interesting nonlinear variations that allow this limit. The method generalizes to Yang-Mills equations and to supersymmetric theories (and may even have application to the theory of tensionless strings). After developing this idea, I shall describe some current explorations of nonlinear electromagnetism respecting conformal symmetry. The conformal compactification of Minkowski space has the topology of $S^1 \times S^3/Z_2$, which maps to the projective light cone in $(4+2)$-dimensional space-time. Here the generators of the conformal group act as rotations. Work going back to Dirac makes use of this idea to discuss linear electrodynamics. But one can write nonlinear constitutive equations that depend on rotation-invariant functionals of the field strength tensor, generalizing the approach to a description of nonlinear conformal electrodynamics. This talk is based on joint work with Vladimir Shtelen and Steven Duplij. (Received January 27, 2012)