The Thin Set Theorem for Pairs implies DNR.

Preliminary report.

Of the Ramsey-like combinatorial principles which have been studied in reverse mathematics, the Thin Set Theorem is among the weakest. The Thin Set Theorem for pairs, TS(2), states that for any coloring function $f : [\omega]^2 \to \omega$ of unordered pairs of integers with (up to) countably many colors, there is an infinite thin set such that $f$ restricted to pairs from $A$ omits at least one color; that is, an infinite set $A$ such that $f([A]^2) \subset \omega$. This principle is a greatly weakened form of Ramsey’s theorem for pairs, and though it is not implied by the base theory RCA$_0$, it has acquired something of a reputation for being uselessly weak. Until recently, TS(2) was not known to imply any “interesting” principles of reverse mathematics. Inspired by a proof that the principle DNR (that for any set $A$ there exists a set diagonally non-computable relative to $A$) is implied by SRT$_2^2$, we show that, in fact, TS(2) implies DNR over RCA$_0$. (Received March 04, 2013)