Let $L(\lambda), L(\mu),$ and $L(\nu)$ be integrable highest-weight representations of $\mathfrak{g} = \widehat{\mathfrak{sl}_2}$ so that $\lambda + \mu + \nu$ is an element of the root lattice. We give a simple condition when $L(N\nu) \subset L(N\lambda) \otimes L(N\mu)$ for $N > 0$ in terms of the weight spaces of $L(\lambda)$ and $L(\nu)$. As a consequence, we show that $L(N\nu) \subset L(N\lambda) \otimes L(N\mu)$ implies that $L(2\nu) \subset L(2\lambda) \otimes L(2\mu)$. We approach the tensor product decomposition problem by computing the characters in terms of string functions and using the Weyl-Kac character formula to arrive at branching functions for $\mathfrak{g} \hookrightarrow \mathfrak{g} \oplus \mathfrak{g}$. We then utilize the action of the Virasoro algebra on $L(\lambda) \otimes L(\mu)$ given by the Sugawara construction, as discussed in [Kac-Wakimoto, Adv. in Math. 70], to interpret these branching functions as characters of unitarizable Virasoro modules. This constrains which $L(\nu)$ do not appear in the decomposition of $L(\lambda) \otimes L(\mu)$ and allows us to arrive at a saturation factor of 2. (Received September 04, 2012)