Jun Tao* (jtao68@yahoo.com), 14207 Eagle Mine Dr, Poway, CA 92064. Proof of the Formula for Arc Length in a New Way.
We approximate the length of a curve using tangent lines that touch the curve at an arbitrary point and get the approximation formula $L \approx \sum_{i=1}^{n} \sqrt{1+\left[f^{\prime}\left(x_{i}^{*}\right)\right]^{2}} \triangle x$, where $x_{i}^{*}$ represents an arbitrary point in the ith subinterval. Then we prove $L=\lim _{n \rightarrow \infty} \sum_{i=1}^{n} \sqrt{1+\left[f^{\prime}\left(x_{i}^{*}\right)\right]^{2}} \triangle x$. Since $x_{i}^{*}$ is an arbitrary point, the proved formula fully satisfies the requirement of the definition of a definite integral and can be converted into the formula $L=\int_{a}^{b} \sqrt{1+\left[f^{\prime}(x)\right]^{2}} d x$. $L=\lim _{n \rightarrow \infty} \sum_{i=1}^{n} \sqrt{1+\left[f^{\prime}\left(x_{i}^{*}\right)\right]^{2}} \triangle x$ is a generic formula and covers the formula $L=\lim _{n \rightarrow \infty} \sum_{i=1}^{n} \sqrt{1+\left[f^{\prime}\left(x_{i}^{c}\right)\right]^{2}} \triangle x$, where $x_{i}^{c}$ represents a certain point, derived by approximating the length of a curve using secant lines. (Received September 22, 2017)

