1135-VP-284Jang Hun Choi*, Roosevelt Hall 309B, Hempstead, NY 11549, and Celine Lee, Soo Hyun
Ahn and Junwon Lee. The Hadwiger-Nelson problem with two forbidden distances.

The chromatic number of the plane problem, named after Hugo Hadwiger and Edward Nelson, asks for the minimum number of colors required to color the plane such that no two points at distance 1 from each other have the same color. It is easy to show that at least four colors are needed, while seven colors are sufficient.

We consider the following related question:

For a given d > 1, what is the minimum number of colors the plane can be colored with such that no two points at distance 1 or d from each other can be assigned the same color?

In this context, we say that 1 and d are forbidden distances. We prove that at least five colors are needed if d takes any of the following values:

 $\sqrt{2}, \sqrt{3}, (\sqrt{5}+1)/2, (\sqrt{6}+\sqrt{2})/2, 2, 2/\sqrt{3}, (2+2\sqrt{3}+2\sqrt{2}\times 31/4)1/2/2$

Our methods are constructive: we find finite point sets in the plane which require five colors if no two points at distance 1 or d from each other can be assigned the same color. We also point out how this problem can lead to a proof of the fact that the chromatic number of the plane is at least 5. (Received August 18, 2017)