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Alexandr Kostochka and **Matthew Yancey*** (yancey1@illinois.edu). *Color-critical graphs with few edges.*

A graph G is k -critical if it has chromatic number k , but every proper subgraph of G is $(k - 1)$ -colorable. Let $f_k(n)$ denote the minimum number of edges in an n -vertex k -critical graph. We give a bound on $f_k(n)$ that is exact for every $n = 1 \pmod{k - 1}$. It is also exact for $k = 4$ and every $n \geq 6$. The result improves the classical bounds by Gallai and Dirac and subsequent bounds by Krivelevich and Kostochka and Stiebitz. It establishes the asymptotics of $f_k(n)$ for every fixed k . We also present some applications of the result, in particular, a simple proof of the Grötzsch Theorem that every triangle-free planar graph is 3-colorable. (Received July 09, 2012)