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Saúl A. Blanco* (sblancor@depaul.edu), Department of Mathematical Sciences, DePaul University, Chicago, IL 60614, and **T. Kyle Petersen**, Department of Mathematical Sciences, DePaul University, Chicago, IL 60614. *Counting Dyck paths by area and rank.*

The set of Dyck paths of length $2n$ inherits a lattice structure from a bijection with the set of noncrossing partitions with the usual partial order. In this paper, we study the joint distribution of two statistics for Dyck paths: *area* (the area under the path) and *rank* (the rank in the lattice).

While area for Dyck paths has been studied, pairing it with this rank function seems new, and we get an interesting (q, t) -refinement of the Catalan numbers. We present two decompositions of the corresponding generating function: one refines an identity of Carlitz and Riordan; the other refines the notion of γ -nonnegativity, and is based on a decomposition of the lattice of noncrossing partitions due to Simion and Ullman.

Further, we show, via Biane's correspondence, that the joint distribution of area and rank for Dyck paths equals the joint distribution of length and reflection length for the permutations lying below the n -cycle $(12 \cdots n)$ in the absolute order on the symmetric group. (Received September 04, 2012)