

1084-20-141

**Peter V Hegarty\*** ([hegarty@chalmers.se](mailto:hegarty@chalmers.se)), Matematiska Vetenskaper, Chalmers University of Technology, Chalmers Tvargata 3, 41296 Gothenburg, Sweden. *Commuting graphs of finite groups.*

The following problem comes from group theory. If  $G$  is a group, the commuting graph of  $G$  has as its vertices the non-central elements of  $G$  and there is an edge between any pair of elements that commute. The concept has been known for over half a century and been studied in a number of contexts. Six years ago, two Iranian mathematicians made the intriguing conjecture that there is an absolute constant  $C > 0$  such that, if  $G$  is a finite group whose commuting graph is connected, then the diameter of the latter cannot exceed  $C$ . The conjecture has attracted considerable attention amongst group theorists, and been verified for a variety of (mostly insolvable) groups. In particular, the largest previously known diameter is 6. Here we will present a family of finite groups which we counter-conjecture to contain examples where the diameter becomes arbitrarily large. The groups in this family are nilpotent of class 2, with both  $Z(G)$  and  $G/Z(G)$  of exponent 2, hence the analysis of their commuting graphs is an "additive problem". A probabilistic method is involved, which thus far provides a clear heuristic, though not a proof, for why arbitrarily large diameters should appear. Simulations have yielded groups where the diameter can attain any value up to 10. Joint work with Dmitry Zhelezov. (Received August 29, 2012)