Ideal transformation optics-based cloaks are sets of singular and anisotropic physical parameters that render an object undetectable to outside observation in both the near- and far-field. In most cases, the cloaking effect is accompanied by shielding, so that external waves do not intrude into the cloaked object. We describe sets of physical parameters (i.e., coefficients of either the Helmholtz or Schrödinger equation) which we call Schrödinger hats. These are obtained by augmenting approximate acoustic or quantum mechanical cloaks with certain parameters inside the cloaked region, giving rise to strong interior resonances. Possible applications include almost cloaked sensors. (Received August 31, 2012)