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Leonid Slavin* (leonid.slavin@uc.edu), Department of Mathematical Sciences, University of Cincinnati, Cincinnati, OH 45221. *Inverse Bellman functions and the exponential integrability of BMO.*

The sharp constants in the John–Nirenberg inequality for BMO,

$$\frac{1}{|Q|} |\{t \in Q : |\varphi(t) - \langle \varphi \rangle_Q| \geq \lambda\}| \leq C_1 e^{-c_0 \lambda / \|\varphi\|_{\text{BMO}}},$$

depend on the choice of the norm; of principal interest is the constant c_0 . For the L^p -based BMO, $c_0 = c_0(p)$ has proved difficult to compute. Until recently, the only results in this direction were by Korenovskii ($c_0(1) = 2/e$) and Vasyunin ($c_0(2) = 1$). The latter result uses Bellman functions; however, even though these functions can be formally defined for any $p > 0$, they cannot be directly computed, unless $p = 2$. This difficulty was recently overcome (so far, for $1 \leq p \leq 2$) by considering the dual problem of estimating (from below) the BMO^p norms of logarithms of A_∞ weights. It turns out that the corresponding Bellman functions are inverses, in an appropriate sense, of those for the original problem. The main result is

$$c_0(p) = \left[\frac{p}{e} \left(\Gamma(p) - \int_0^1 t^{p-1} e^t dt \right) + 1 \right]^{1/p}, \quad 1 \leq p \leq 2.$$

The proof relies on finding optimal convex solutions of the homogeneous Monge–Ampère equation on a non-convex plane domain. (Received September 02, 2012)