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David V. Cruz-Uribe* (david.cruzuribe@trincoll.edu), Department of Mathematics,
Trinity College, 300 Summit St., Hartford, CT 06110. *The rise, fall and rebirth of the
Muckenhoupt-Wheeden conjecture.*

Muckenhoupt and Wheeden conjectured that given weights u, v and $1 < p < \infty$, the Hilbert transform satisfies $H : L^p(v) \rightarrow L^p(u)$ if the maximal operator satisfies $M : L^p(v) \rightarrow L^p(u)$ and the “dual” inequality $M : L^{p'}(u^{1-p'}) \rightarrow L^{p'}(v^{1-p'})$. They also conjectured that the dual inequality was sufficient for the weak (p, p) inequality $H : L^p(v) \rightarrow L^{p,\infty}(u)$. While very attractive, these conjectures are false: this was proved for the strong-type inequality by Reguera and Scurry, and for the weak-type by us with Reznikov and Volberg. On the other hand, we proved with Martell and Pérez that this conjecture is true for L^p, L^q inequalities when $p < q$. And, working with Moen, we proved that a related conjecture for Riesz potentials is also true when $p < q$. We will discuss these results and explore their connection with the A_p bump conditions. These were introduced by Pérez in the 1990s, motivated by the Muckenhoupt-Wheeden conjecture. We give a new condition that was implicit in his work, and describe a partial result in the scale of log-bumps, joint with Volberg and Reznikov. We will also discuss a related theorem for Riesz potentials from work with Moen. (Received September 03, 2012)