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**Tamas Darvas\*** (tdarvas@math.purdue.edu), Department of Mathematics, Purdue University, 150 N. University Street, West Lafayette, IN 47907. *Morse theory and geodesics in the space of Kähler metrics.*

Given a compact Kähler manifold  $(X, \omega_0)$  let  $\mathcal{H}_0$  be the set of Kähler forms cohomologous to  $\omega_0$ . As observed by Mabuchi, this space has the structure of an infinite dimensional Riemannian manifold, if one identifies it with a totally geodesic subspace of  $\mathcal{H}$ , the set of Kähler potentials of  $\omega_0$ . Following Donaldson's research program, existence and regularity of geodesics in this space is of fundamental interest. In this paper, supposing enough regularity of a geodesic  $u : [0, 1] \rightarrow \mathcal{H}$ , connecting  $u_0 \in \mathcal{H}$  with  $u_1 \in \mathcal{H}$ , we establish a Morse theoretic result relating the critical points of  $u_1 - u_0$  to the critical points of  $\dot{u}_0 = du/dt|_{t=0}$ . As an application of this result, we prove that on all Kähler manifolds, connecting Kähler potentials with smooth geodesics is not possible in general. In particular, in the case  $X \neq \mathbb{C}P^1$ , we will also prove that the set of pairs of potentials that can not be connected with smooth geodesics has nonempty interior. (Received August 23, 2012)