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*Computable and Incomputable Boundary Extensions of Conformal Maps.*

Suppose  $\phi$  is a conformal map on the unit disk (that is, the open disk whose center is the origin and whose radius is 1) onto a domain  $D$ . If  $\phi$  has a continuous extension to the closed unit disk, then this extension is referred to as the *boundary extension* of  $\phi$ . A well-known theorem states that the boundary extension of  $\phi$  exists if and only if the boundary of  $D$  is bounded and locally connected. In addition, if the boundary of  $D$  is a Jordan curve, then the boundary extension of  $\phi$  is a homeomorphism. We show that one direction of this theorem holds effectively in that if  $\phi$  is computable and if the boundary of  $D$  is effectively locally connected, then the boundary extension of  $\phi$  is computable. However, we show that the other direction fails effectively in that there is a computable conformal map that has a boundary extension even though the boundary of its range is not effectively locally connected. Furthermore, we show that there is a computable conformal map from the unit disk onto a Jordan domain whose boundary extension is incomputable. Altogether, these results say that effective local connectivity provides sufficient, non-superfluous, but excessive information for the computation of boundary extensions. (Received July 23, 2013)