We consider the descriptive complexity of some subsets of the infinite permutation group $S_\infty$ which arise naturally from the classical series rearrangement theorems of Riemann, Levy, and Steinitz. In particular, given some fixed conditionally convergent series of vectors in Euclidean space $\mathbb{R}^d$, we study the set of permutations which make the series diverge, as well as the set of permutations which make the series diverge properly. We show that both collections are $\Sigma_3^0$-complete in $S_\infty$, regardless of the particular choice of series.

The proof involves a blend of the descriptive set theoretic notion of continuous reducibility, with the purely geometric techniques employed by Steinitz himself in his original proof of the Levy-Steinitz Theorem. In particular, we appeal to the existence of a perhaps not sufficiently famous geometric constant, now referred to as the Steinitz constant. (Received November 07, 2012)