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V. Mykhaylyuk, M. Popov and B. Randrianantoanina*, randrib@muohio.edu, and **G. Schechtman**. *Narrow and ℓ_2 -strictly singular operators from L_p .*

Let X be a Banach space. A linear operator $T : L_p \rightarrow X$ is called narrow if for each $\varepsilon > 0$ and each measurable set $A \subseteq [0, 1]$ there exists $x \in L_p$ with $x^2 = \mathbf{1}_A$, $\int_{[0,1]} x d\mu = 0$, so that $\|Tx\| < \varepsilon$. It is easy to see that compact operators are narrow, however, in general, the class of narrow operators is much larger than that of compact operators.

We prove that for $2 < p, r < \infty$ every operator $T : L_p \rightarrow \ell_r$ is narrow; and that every ℓ_2 -strictly singular operator from L_p , $1 < p < \infty$, to any Banach space with an unconditional basis, is narrow, which partially answers a question of Plichko and Popov posed in 1990.

H.P. Rosenthal proved that an operator T on $L_1[0, 1]$ is narrow if and only if for each measurable set $A \subseteq [0, 1]$ the restriction $T|_{L_1(A)}$ is not an isomorphic embedding. Inspired by this result, we find a sufficient condition, of a different flavor than being ℓ_2 -strictly singular, for operators on $L_p[0, 1]$, $1 < p < 2$, to be narrow. We define a notion of a “gentle” growth of a function and prove that for $1 < p < 2$ each operator T on L_p which, for any $A \subseteq [0, 1]$, sends a function of “gentle” growth supported on A to a function of arbitrarily small norm is narrow. (Received December 04, 2012)