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A *generalized hoop*  $(A, \cdot, 1, \backslash, /)$  is a residuated partially ordered monoid in which  $x \leq y$  iff  $\exists u(x = uy)$  iff  $\exists v(x = yv)$ . Generalized hoops form a congruence distributive variety defined by, e.g.,  $x1 = x$ ,  $x \backslash x = 1 = x/x$ ,  $(xy) \backslash z = y \backslash (x \backslash z)$ ,  $x/(yz) = (x/z)/y$ ,  $(x/y)y = y(y \backslash x) = (y/x)x$ . The term  $(x/y)y$  defines a meet operation, hence is denoted by  $x \wedge y$ .

An *integral GBL-algebra* is a generalized hoop expanded with a join operation  $\vee$  such that  $(A, \wedge, \vee)$  is a lattice. Examples of such algebras are given by Brouwerian algebras and negative cones of  $\ell$ -groups. A *hoop* is a commutative generalized hoop. We give a description of the HS-poset of all finite subdirectly irreducible generalized hoops, from which the lattice of finitely generated varieties of generalized hoops can be constructed as the lattice of downward closed subsets. A similar description for integral GBL-algebras is also given.

We show that the equational theory of basic hoops and BL-algebras can be decided efficiently using SMT-solvers. We also discuss the problem of whether generalized hoops have a decidable equational theory. (Received February 19, 2013)