We study the collapse of the nonlinear Schrödinger equation (NLS) in critical case of dimension two. The collapse describes e.g. self-focusing of light in nonlinear Kerr media. The scaling of self-similar solutions near collapse point has \((t_0 - t)^{1/2}\) scaling law with the logarithmic modifications of log-log type. We show that the well-known leading order log-log modification occurs for nonrealistic exponentially large amplitudes of light \(\sim 10^{10^{100}}\). Instead we derived a new equation for adiabatically slow parameter which determines the system dynamics. Based on that equation we develop a perturbation theory for scaling modifications beyond leading log-log order and perform detailed comparison with simulations. We show that new scaling agrees with simulations for very moderate increase (\(\sim 3\) times) of the amplitude of initial pulse. (Received January 18, 2013)