Partial differential equations posed on surfaces arise in mathematical models for many natural phenomena: diffusion along grain boundaries, lipid interactions in biomembranes, pattern formation, and transport of surfactants on multiphase flow interfaces to mention a few. Numerical methods for solving PDEs posed on manifolds recently received considerable attention. In this talk we briefly review some existing approaches and focus on an Eulerian finite element method for the discretization of elliptic and parabolic partial differential equations on surfaces. The method uses traces of volume finite element space functions on a surface to discretize equations posed on the surface. The approach is particularly suitable for problems in which the surface is given implicitly by a level set function and in which there is a coupling with a problem in a fixed outer domain. If the surface evolve, then the method employs DG space-time finite elements for a space-time weak formulation of a surface PDE problem. In this case, trial and test surface finite element spaces consist of traces of standard volumetric elements on a space-time manifold resulting from the evolution of a surface. (Received January 17, 2014)