Embedding high-dimensional data sets into subspaces of much lower dimension is important for reducing storage cost and speeding up computation in several applications, including numerical linear algebra, manifold learning, and theoretical computer science. Moreover, central to the relatively new field of compressive sensing, if the original data set is known to be sparsely representable in a given basis, then it is possible to efficiently ’invert’ a random dimension-reducing map to recover the high-dimensional data via e.g. $l_1$-minimization. We will survey recent results in these areas, and then show how near-equivalences between fundamental concepts such as restricted isometries and Johnson-Lindenstrauss embeddings can be used to leverage results in one domain and apply to another. Finally, we discuss how these and other recent results for structured random matrices can be used to derive sampling strategies in various settings, from low-rank matrix completion to function interpolation. (Received February 11, 2014)