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Jeremy F Alm* (alm.academic@gmail.com), 1101 W. College Ave., Jacksonville, IL 62650, and
Jacob Manske, Verona, WI 53593. *Highly constrained edge-colorings of complete graphs, part I.*

Given a finite set \mathcal{C} of colors, let \mathcal{T} be a set of triangles with edges colored from colors in \mathcal{C} , i.e., $\mathcal{T} \subseteq \mathcal{C}^3$ and \mathcal{T} is closed under permutation of coordinates. \mathcal{T} is *realizable* if there is some $N \leq \omega$ such that there exists an edge-coloring of K_N with colors from \mathcal{C} such that

- every triangle appearing in K_N is in \mathcal{T} , and
- if uv is colored a and $\langle a, b, c \rangle \in \mathcal{T}$, then there is a vertex w such that uw is colored b and vw is colored c , i.e., “Whatever can happen, must happen”.

Let $\mathcal{C} = \{a_1, \dots, a_n\}$, and let $\mathcal{T}_m = \{\langle a_i, a_j, a_k \rangle : |\{i, j, k\}| \neq m\}$, ($m = 1, 2, 3$). Then

- It is an open problem whether \mathcal{T}_1 is realizable for all n . Erdős, Szemerédi, and Trotter once gave a purported proof, but it was wrong.
- If \mathcal{T}_2 is realizable on K_N , then there is a projective plane of order $n - 1$.
- If $\mathcal{T} \subseteq \mathcal{T}_3$, then it is decidable whether \mathcal{T} is realizable, realizable over finite sets only, etc.

In Part II, Manske will discuss some recent progress on open problems. (Received July 29, 2014)