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Marcos César Vargas Magaña* (e_socram@math.cinvestav.edu.mx) and **Carlos E. Valencia Oleta** (cvalencia@math.cinvestav.edu.mx). *Optimum perfect matchings in a weighted bipartite graph*. Preliminary report.

Given a bipartite graph $G = (X, Y; E)$, a *matching* is a subset of disjoint edges M of G . Moreover, it is called *perfect* if it covers all the vertices of G . Given a cost function $c : E \rightarrow \mathbb{Z}$, the cost of M is given by the sum of the costs of its edges. The *assignment problem* consists of finding a perfect matching M of optimum (maximum or minimum) cost.

Let $f : E \rightarrow [0, 1]$ and $p : X \cup Y \rightarrow \mathbb{R}$ be the primal and the dual variables of the linear program associated to the assignment problem on G with cost function c . A pair $\{f, p\}$ is called optimum if f and p are optimum solutions of the primal and dual linear programs. Let $G_{opt} = (X \cup Y, E_{opt})$ the subgraph of G with edge set

$$E_{opt} = \{e \in E \mid e \text{ is in a perfect matching of optimum cost of } G\}.$$

Given an optimum pair $\{f, p\}$ we present an algorithm that finds G_{opt} in time $O(|E|)$. This algorithm can be complemented with any algorithm that finds an optimum pair $\{f, p\}$. For instance, adapting the ϵ -scaling auction algorithm of Bertsekas we developed an algorithm that finds G_{opt} from the pair $\{G, c\}$ in time $O(|X||E| \log(|X|C))$, where C is the maximum absolute cost. (Received May 03, 2013)