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Given a graph  $G = (V, E)$  with  $n$  vertices, the *generalized Laplacian matrix* of  $G$  is

$$L(G, X_G)_{u,v} = \begin{cases} x_u & \text{if } u = v, \\ -m_{(u,v)} & \text{otherwise,} \end{cases}$$

where  $m_{(u,v)}$  is the number of edges between vertices  $u$  and  $v$ . If  $d_G$  is the degree vector of  $G$ , then  $L(G, X = d_G)$  is the Laplacian matrix  $L(G)$  of  $G$ . The *critical ideals* of  $G$  are given by

$$I_i(G, X_G) = \langle \text{minors}_i(L(G, X_G)) \rangle \subseteq \mathcal{R}[X_G] \text{ for all } 1 \leq i \leq n,$$

where  $\mathcal{R}$  is a commutative ring. Let  $\gamma_{\mathcal{R}}(G) = \max\{i \mid I_i(G, X) = \langle 1 \rangle\}$ . We say that a graph is  *$\gamma$ -critical* when  $\gamma_{\mathcal{R}}(G \setminus v) < \gamma_{\mathcal{R}}(G)$  for all  $v \in V$ . Moreover, when  $\mathcal{R} = \mathbb{Z}$ , critical ideals generalize the critical group of a graph; the torsion part of the *cokernel* of  $L(G)$ . The study and characterization of graphs with critical group having  $i$  invariant factors equal to one have been of great interest. We classify all the  $\gamma$ -critical graphs with  $\gamma \leq 2$ . As consequence we get a classification of the simple graphs whose critical group has one and two invariant factors equal to one. (Received May 08, 2013)