Smooth curves in the tropical plane correspond to unimodular triangulations of lattice polygons. The skeleton of such a curve is a metric graph whose genus is the number of lattice points in the interior of the polygon. In this lecture we report on work concerning the following realizability problem: Characterize all metric graphs that admit a planar representation as a tropical curve. For instance, 20 percent of metric graphs with genus 3 have this property. (Received June 04, 2014)