The Berkovich skeletons of spaces of stable maps are combinatorially rich objects, that interact intricately with tropical geometry and degeneration techniques. In this talk, I will report on recent and ongoing work with Renzo Cavalieri and Hannah Markwig in which we study spaces of maps to target curves. The skeleton of the analytification of spaces of relative stable maps and admissible covers admit (tropical) modular interpretations. These interpretations allow us to use combinatorial methods to compute certain relative Gromov–Witten invariants and Hurwitz numbers. In the genus 0 case, we use these techniques to study tropicalizations of double Hurwitz loci, and explain piecewise polynomiality properties of the tropical Hurwitz cycle. (Received August 20, 2014)