This work is in joint collaboration with Professor Thomas Sideris (UCSB). We consider small-data solutions to equations of the form
\[
\begin{align*}
\Box u(t, x) &= Q(\partial u, \partial^2 u), \\
        u(0, x) &= \epsilon f(x), \\
    \quad \partial_t u(0, x) &= \epsilon g(x),
\end{align*}
\]
where $\Box = \partial_t^2 - \Delta$ and the nonlinearity $Q$ is allowed to depend on $\partial u$ and $\partial^2 u$ at the quadratic level and higher. We also assume that $Q$ is linear in $\partial^2 u$ and that $Q$ satisfies a null condition, which is due to Christodoulou and Klainerman. Alinhac proved global existence of small-data solutions with smooth, compactly supported data $(f, g)$ by using a “Ghost weight” in his main energy estimate. Our proof extends Alinhac’s result by allowing for a weaker hypothesis on the initial data: $(f, g)$ are only required to have a certain amount of weighted Sobolev regularity with no restrictions on the support. Our proof also eliminates the use of the Lorentz boosts $x_i \partial_t + t \partial_i$ ($i = 1, 2$) from the existence argument. If time permits, I will also discuss some difficulties present in this problem and some possible follow-up projects. (Received September 01, 2014)