We prove dispersive estimates for the Schrödinger evolution $e^{itH}P_{ac}(H)$ in $\mathbb{R}^n$, $n \geq 5$, where $H = -\Delta + V(x)$ has an eigenvalue at zero. As a map from $L^1(\mathbb{R}^n)$ to $L^\infty(\mathbb{R}^n)$ there is a rank one term decaying at the rate $|t|^{2-\frac{n}{2}}$ and a finite rank operator with time decay $|t|^{1-\frac{n}{2}}$. The asymptotic expansion continues into more heavily weighted spaces; we show in particular that the remainder term after these finite rank pieces exists as a map from $\langle x \rangle^{-2}L^1$ to $\langle x \rangle^2L^\infty$.

The initial finite rank terms both vanish if the eigenspace of $H$ satisfies certain cancellation conditions, or equivalently if each eigenfunction satisfies the bound $|\psi(x)| \leq C(1 + |x|)^{-n}$. The extra cancellation also removes all need for weights when describing the behavior of the remaining evolution. Under those conditions we recover the same dispersive bound, mapping $L^1(\mathbb{R}^n)$ to $L^\infty(\mathbb{R}^n)$ with norm $|t|^{-\frac{n}{2}}$, as when zero is a regular point of the spectrum. (Received September 02, 2014)