We propose algebraic criteria that yield sharp Hölder type of inequalities for the product of functions of Gaussian random vectors with arbitrary covariance structure. While our lower inequality appears to be new, we prove that the upper inequality gives an equivalent formulation for the geometric Brascamp-Lieb inequality for Gaussian measures. As an application, we retrieve the Gaussian hypercontractivity as well as its reverse and we present a generalization of the sharp Young and reverse Young inequalities. From the latter, we recover several known inequalities in literature including the Prékopa-Leindler and Barthe inequalities. (Received September 03, 2014)