We study parameter estimation in linear Gaussian covariance models, which are $p$-dimensional Gaussian models with linear constraints on the covariance matrix. Maximum likelihood estimation for this class of models leads to a non-convex optimization problem which typically has many local optima. We prove that the log-likelihood function is concave over a large region of the cone of positive definite matrices. Using recent results on the asymptotic distribution of extreme eigenvalues of the Wishart distribution, we provide sufficient conditions for any hill climbing method to converge to the global optimum. The proofs of these results utilize large-sample asymptotic theory under the scheme $n/p \to \gamma > 1$. Remarkably, our numerical simulations indicate that our results remain valid for $\min\{n, p\}$ as small as 2. An important consequence of this analysis is that for sample sizes $n \simeq 14p$, maximum likelihood estimation for linear Gaussian covariance models behaves as if it were a convex optimization problem.

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