A lattice polytope $P \subset \mathbb{R}^n$ is the convex hull of finitely many points in $\mathbb{Z}^n$. There is a natural hierarchy of structural sophistication for lattice polytopes, with various concepts motivated from toric geometry and commutative algebra. We will discuss three such concepts in this hierarchy, occupying a point of origin (normality), the bottom (very ampleness), and the top spot (Koszul property). More specifically, we explore a simple construction for lattice polytopes with a twofold aim. On the one hand, we derive an explicit series of very ample 3-dimensional polytopes with arbitrarily large deviation from the normality property, measured via the highest discrepancy degree between the corresponding Hilbert functions and Hilbert polynomials. On the other hand, we describe a large class of Koszul polytopes of arbitrary dimensions, containing many smooth polytopes and extending the previously known class of Nakajima polytopes. (Received January 12, 2014)