Paul-Jean Cahen, David E Dobbs and Thomas G Lucas* (tglucas@uncc.edu). Valuative Pairs of Rings.

An integral domain $D$ is valuative if for each $x \in qf(D) \setminus \{0\}$, at least one of $D \subseteq D[x]$ and $D \subseteq D[1/x]$ has no proper intermediate rings. We extend this notion to pairs of commutative rings $R \subseteq T$ with a common nonzero identity: the ring $R$ is $T$-valuative if for each $t \in T \setminus \{0\}$, at least one of $R \subseteq R[t]$ and $R \subseteq R[(R :_T t)]$ has no proper intermediate rings. There are weak and strong versions as well: $R$ is weakly $T$-valuative if for each $t \in T$, either $R \subseteq R[t]$ has no proper intermediate rings or $R \subseteq R[s]$ has no proper intermediate rings for each $s \in (R :_T t)$; $R$ is strongly $T$-valuative if for each $t \in T$, at least one of $R \subseteq R[(R :_T (R :_T t))]$ and $R \subseteq R[(R :_T t)]$ has no proper intermediate rings. In general, the following implications are not reversible: strongly $T$-valuative $\Rightarrow$ $T$-valuative $\Rightarrow$ weakly $T$-valuative (even when $T$ is the total quotient ring of $R$). However if $R$ is integrally closed in $T$, then all three are equivalent. Unlike valuative domains (which have at most three maximal ideals), (strongly) [weakly] $T$-valuative rings can have infinitely many maximal ideals. (Received January 24, 2014)