A power series that converges on the unit disc $\mathbb{D}$ is called universal if its partial sums approximate arbitrary polynomials on arbitrary compacta in $\mathbb{C}\setminus\mathbb{D}$ that have connected complement. In this talk we will show that such series grow strongly and possess a Picard-type property near each boundary point. (Received January 11, 2014)