In this talk we consider spectral, inverse spectral and inverse scattering for the differential equation
\[-u'' + qu = \lambda w u\]

where \( q \geq 0 \) but where \( w \) is only required to be real and locally integrable but not positive.

Applications include solving the Cauchy problem for the Camassa-Holm equation. (Received January 22, 2014)