Finite energy solutions of quasilinear elliptic equations with sub-natural growth terms.

We study finite energy solutions to quasilinear elliptic equations of the type

\[-\Delta_p u = \sigma u^q \text{ in } \mathbb{R}^n,\]

where \(\Delta_p\) is the \(p\)-Laplacian, \(p > 1\), and \(\sigma\) is a nonnegative function (or measure) on \(\mathbb{R}^n\), in the case \(0 < q < p - 1\) (below the “natural growth” rate \(q = p - 1\)). We give explicit necessary and sufficient conditions on \(\sigma\) which ensure that there exists a solution \(u\) in the homogeneous Sobolev space \(L^{1,p}_0(\mathbb{R}^n)\), and prove its uniqueness. Among our main tools are integral inequalities closely associated with this problem, and Wolff potential estimates used to obtain sharp bounds of solutions. More general quasilinear equations with \(\text{div} A(x, \nabla u)\) in place of \(\Delta_p u\) are considered as well. This is joint work with Igor E. Verbitsky. (Received January 20, 2014)